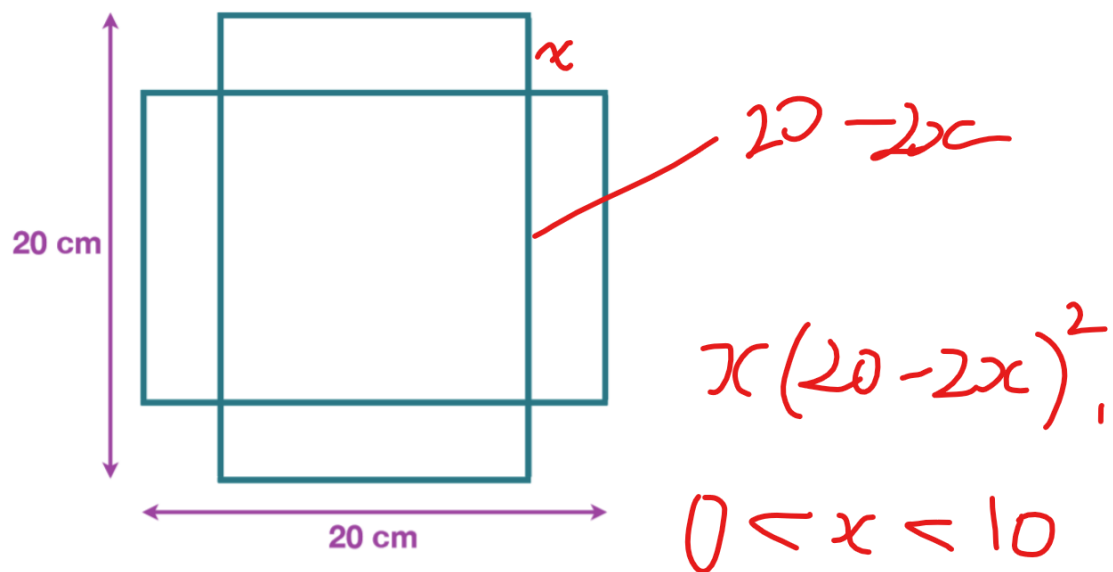




For a box generated this way, the height is x , and the side lengths of the square are $20 - 2x$. The volume is $x(20 - 2x)^2$.

$$\begin{aligned}
 &= x(400 - 80x + 4x^2) \\
 &= 4x^3 - 80x^2 + 400x \\
 &= 4x(x^2 - 20x + 100)
 \end{aligned}$$

Also, the domain of the function is $0 < x < 10$.

1. Using AM-GM

The AM-GM equality states that $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$ for real numbers a , b , and c . We can set them to cancel out like this:

$$\begin{aligned}
 a &= x \\
 b &= \frac{10 - x}{2} \\
 c &= \frac{10 - x}{2} \\
 \frac{a + b + c}{3} &\geq \sqrt[3]{abc} \\
 abc &= \frac{4x(10 - x)^2}{3} \\
 a + b + c &= 10 \\
 \sqrt[3]{\frac{x(10 - x)^2}{4}} &\leq \frac{10}{3} \\
 \frac{x(10 - x)^2}{4} &\leq \frac{10^3}{3^3} \\
 \frac{100x - 20x^2 + x^3}{4} &\leq \frac{1000}{27}
 \end{aligned}$$

$$4(100x - 20x^2 + x^3) \leq \frac{16000}{27}$$

2. Using Graphical

We can simply plot the graph of $y = x(20 - x)^2$ between 0 and 10 to find the maximum of the value, using a GDC or any graphing calculator. We find this value is $x = 3\frac{1}{3}$ or $\frac{10}{3}$, plugging it back in we get $\frac{16000}{27}$ for the volume.